INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATIONS-June/July 2019

B.Tech(Marine Engineering)

Semester-II

Mathematics-II (UG11T1202 / UG11T2202)

Date: 27-06-2019 **Maximum Marks: 100**

Duration: 3 hrs **Pass Marks: 50**

i. Use of approved type of scientific calculator is permitted
ii. The symbols have their usual meaning

Part - A **(3x10=30)**

(All Questions are Compulsory)

Q1(a). Find b_n of the Fourier series for the following function

$$f(x) = -\pi, \quad -\pi < x < 0$$
$$= x, \quad 0 < x < \pi$$

(b). Find the Laplace transform of $L[\int_0^t e^t \frac{\sin t}{t} dt]$.

(c). Find the Laplace transform of $L\{\int_0^t\int_0^t\int_0^t (t\sin t)dtdtdt\}$.

(d). Use step function to evaluate the Laplace transform of the

Following function $f(t) = \begin{cases} t, 0 < t < 2 \\ 2, t > 2 \end{cases}$

(e). Find only the integrating factor for the non-exact equation

$(x^2y^3 - y)dx + (x^3y^2 + x)dy = 0$.

(f). Find the orthogonal trajectory of the family of curves $r^n = a\sin n\theta$ (a is the parameter).

(g). Solve $x^4 \frac{dy}{dx} + x^3 y + \cos ec(xy) = 0$.

(h). Given P(A)=1/4, P(B)=1/3 and P(A$\cup$B)=1/2. Evaluate

P(A\B),P(B\A),P(A$\cap$B) and P(A\B′)

(i). The diameter of an electric cable is assumed to be a continuous

variate f with possible probability density function

(x)=6x(1-x), $0 \le x \le 1$

Verify that f is a probability density function. Also find the mean and variance.

(j).One hundred litres of water have been polluted with 10^6

bacteria. If a sample of 1c.c of water is taken, show that the probability it is not polluted is e^{-10}

PART B **(5x14=70 Marks)**

(Answer any five questions)

2(a). Find a series of cosines of multiples of x which will represent

x sin x in the interval $-\pi \le x \le \pi$ and show that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi - 2}{4}$$ (7)

2(b). Find the Fourier sine series for unity in $(0 < x < \pi)$ and hence

show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = \frac{\pi^2}{8}$ (7)

3(a). Find the inverse Laplace transform of $\frac{s}{s^4 + s^2 + 1}$ (5)

3(b). Evaluate $\int_0^\infty t^3 e^{-t} \sin t\,dt$, (∞= infinity) (4)

3(c). Apply convolution theorem to evaluate $L^{-1}\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\}$ (5)

4(a). Solve $3e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$ (4)

4(b). Using method of variation of parameters solve

$$\frac{d^2y}{dx^2} + 4y = \tan 2x \qquad (4)$$

4(c). Solve $\frac{d^2y}{dx^2} + y = \sec^2 x$ (6)

5(a). Two boxes contain respectively 4 white and 2 black and 1 white And 3 black balls. One ball is transferred from the first box into the second and then one ball is drawn from the latter. It turns out to be black. What is the probability that the transferred ball was white. (7)

5(b). The probability density function of the random variable X follows Probability law $P(x) = \frac{1}{2\theta}\exp(-\frac{|x-\theta|}{\theta})$, $-\infty < x < \infty$

Find Moment Generation Function of X. Hence or otherwise find E(X) and Var(X). (7)

6(a). The probability of a man hitting a target is ¼.

(i) If he fires 7 times what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3. (7)

6(b). Prove that the binomial distribution the following relation holds good $\mu_{r+1} = pq(\frac{d\mu_r}{dp} + nr\mu_{r-1})$ (7)

7(a). Find the Fourier series of x^2 in $(-\pi, \pi)$. Use perseval's identity to prove that $\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (7)

7(b). Using the method of undetermined coefficients solve

$$(D^3+3D^2+2D)y=x^2+4x+8 \qquad (7)$$

8(a). Using Laplace transform solve the equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x=e^{-t}\sin t\,,\ \ x(0)=0, x'(0)=1 \qquad (7)$$

8(b). A voltage Ee^{-at} is applied at t=0 to a circuit of inductance L and resistance R. Show that the current at time t is

$$\frac{E}{R-aL}(e^{-at}-e^{\frac{-Rt}{L}}) \qquad (7)$$